

Temperature dependent resistivity of spin-split subbands in GaAs 2D hole system

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We calculate the temperature dependent resistivity in spin-split subbands induced by the inversion asymmetry of the confining potential in GaAs 2D hole systems. By considering both temperature dependent multisubband screening of impurity disorder and hole-hole scattering we find that the strength of the metallic behavior depends on the symmetry of the confining potential (i.e., spin-splitting) over a large range of hole density. At low density above the metal-insulator transition we find that effective disorder reduces the enhancement of the metallic behavior induced by spin-splitting. Our theory is in good qualitative agreement with existing experiments.

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The observation of an apparent “metallic” behavior in low density two dimensional (2D) electron or hole semiconductor systems has attracted a great deal of recent attention [1] because of the long-standing belief that all 2D (disordered) electronic systems (at least in the noninteracting limit) are insulators (i.e., infinite zero temperature resistance in the thermodynamic limit). Although the experimental situation is controversial (to say the least) because the “metallicity” of the 2D system must invariably be inferred from an extrapolation (to zero temperature) of finite temperature (usually $T \geq 100mK$) data, there is broad agreement on a number of intriguing and manifestly interesting features of the observed 2D resistivity, $\rho(T, n)$, as a function of temperature (T) and 2D free carrier density (n), that require theoretical understanding irrespective of whether there is a true $T = 0$ 2D metal or not.

One of the most striking experimental anomalies in this subject is the strong temperature dependence of the resistivity of this effective 2D conducting phase at low carrier densities and in the relatively modest temperature range of $0.1K - 4K$ where the measured resistivity may change by as much as a factor of three whereas the corresponding resistivity change in this temperature range (the so-called Bloch-Grüneisen behavior) for a real 3D metal (e.g., Cu, Al) is minuscule (less than 1%). In fact, very weak temperature dependence of resistivity is also observed in 2D electron systems at higher densities, but at lower densities (and in relatively clean samples) $\rho(T)$ shows strikingly strong temperature dependence, not found in any known metallic systems. This very strong dependence of $\rho(T, n)$ on T at low n , specifically in 2D p-GaAs hole systems, is the subject matter of this paper.

There have been two alternate concrete physical mechanisms proposed in the literature to explain the strong temperature dependence of $\rho(T, n)$ in the 2D “metallic” systems. One mechanism [2] is the possible temperature dependence of effective impurity disorder in the system, arising, for example, from the temperature dependence of finite wave vector 2D screening which could be very strong at low carrier densities. This mechanism [2-4] is

quite universal in the sense that all 2D carrier systems should manifest some aspects of temperature dependent screening since random (unintentional) charged impurities (and often, particularly in modulation doped 2D systems, remote dopants as well) are invariably present in semiconductors. We have earlier shown through detailed transport calculations [2] that the temperature dependent disorder scattering could in fact provide a reasonable semi-quantitative explanation for the observed temperature dependence of 2D resistivity behavior in n-Si MOSFETs, p-SiGe, and p-GaAs 2D systems. The second mechanism for the temperature dependence of resistivity, which applies essentially only to 2D p-GaAs hole systems in a quantitatively significant manner, has been discussed extensively by the Princeton group and others [5-7]. This manifestly non-universal mechanism involves a temperature dependent transport mechanism arising from intersubband hole scattering between two spin-split hole bands, which are present in the GaAs valence band. The magnitude of this effect should therefore correlate with the valence band spin-splitting as has been demonstrated experimentally. This proposal has been somewhat controversial, and there are experimental claims (and counter-claims) for nonexistence (and the existence) of this spin-splitting induced intersubband scattering mechanism. It is obvious that both of these physical mechanisms must be simultaneously operational in the p-GaAs system because scattering by random charged impurities and valence band spin-splitting are both unavoidably present in real 2D GaAs hole structures. What is therefore required is a (technically highly demanding) theory which includes both mechanisms on an equal footing, and in this paper we provide such a theory which, as we describe below, qualitatively explains all of the observed features of the temperature dependent $\rho(T, n)$ of 2D GaAs hole systems. Our work convincingly shows that in order to understand the 2D GaAs hole data comprehensively a theory such as ours including both mechanisms equivalently is necessary although one mechanism or the other may very well be dominant in specific experimental p-GaAs systems depending on

the details of disorder and spin-splitting (which would vary from sample to sample), thus resolving the existing controversy of whether spin-splitting is a relevant mechanism in 2D “metallic” behavior or not. In all our discussions in this paper, consistent with the accepted (but, not quite rigorous) terminology in this subject, the word 2D “metal” or “metallic” behavior will only mean a strong temperature dependence of $\rho(T)$ at a fixed density (above the so-called metal-insulator transition “critical” density), with $d\rho/dT$ being positive at low temperatures.

The valence band holes in GaAs-AlGaAs heterostructures have large spin-orbit interaction due to the inversion asymmetry of the zincblende crystal structure and the real space asymmetry induced by the confining potential, producing substantial zero-magnetic-field spin-splitting [8]. The lack of inversion symmetry in the p-GaAs system can lift the spin degeneracy and produce two groups of spin-split 2D subbands of holes with different band dispersions and transport properties. Recently, Papadakis *et al.* [5] have experimentally studied the relationship between the 2D metallic behavior and the spin-splitting of the p-GaAs quantum well systems by tuning the symmetry of the well confinement potential while keeping the total hole density constant. They find that over a large range of densities (in a regime where the samples exhibit metallic behavior) the symmetry of the quantum well plays an important role in the sense that the temperature dependence of $\rho(T)$ seems to correlate with the (well asymmetry tuned) spin-splitting. At high hole densities, far above the critical density for the apparent metal-insulator transition (MIT), the metallic behavior is very pronounced as the spin-splitting becomes large, and the two spin-split subbands are occupied with unequal densities. This enhancement of the metallic behavior is induced by the temperature dependent intersubband hole-hole scattering because at constant total density the intersubband scattering becomes stronger as the spin-splitting increases. Thus, the temperature dependent hole-hole scattering may, by itself, give rise to the apparent metallic behavior in the spin-split situation, at least for large carrier densities. In low density samples [5,6], however, the spin-splitting becomes much smaller, and the change of the resistivity with temperature arising from intersubband hole-hole scattering is suppressed even in the presence of large asymmetry in the confining potential. Strong metallic behavior of $\rho(T)$ at low densities cannot therefore be explained solely by spin-splitting effects. This indicates that additional scattering mechanisms other than hole-hole scattering are required to explain the metallic behavior at low densities. Hamilton *et al.* [6] claim, based on the interpretation of their own 2D GaAs hole data, that in the low density systems the metallic behavior is determined not by the spin-splitting induced intersubband carrier-carrier scattering, but by the magnitude of the low temperature resistivity, that is, by the effective disorder in the system.

In this paper we investigate the 2D hole metallic behavior observed in p-type GaAs systems including both disorder and spin-splitting effects. This system forms two spin-split hole subbands (without any band offset) with very different effective masses. Our goal is to calculate the temperature dependent resistivity in the spin-split two subband system. We use the Boltzmann transport equation approach to calculate the low temperature ohmic resistivity of the two subband system taking into account the single particle relaxation times of each subband (as arising from charged impurity scattering), τ_i , and the hole-hole intersubband scattering between the two groups of holes, τ_{hh} , which relaxes the relative momentum of the two carriers [9,10] and therefore contributes to the net resistivity of the two subband system. We calculate the single particle relaxation times τ_i from a multisubband transport theory taking into account the long range Coulombic scattering by random static charged impurity centers invariably present in semiconductor structures. Since screening plays a crucial role in the temperature dependence of resistivity [2] we consider both hole-impurity and hole-hole Coulomb interactions being screened on an equal footing by the 2D hole gas in the random phase approximation (RPA) using a multisubband screening formalism. In the multisubband system screening is enhanced since all subbands contribute. The screening effect can, in the RPA, be included in terms of a matrix dielectric formalism [11,12], $\epsilon_{ij,i'j'}(q) = \delta_{ij}\delta_{i'j'} - v(q)\Pi_{ij}(q)F_{ij,i'j'}(q)$, where i, j are the band indices, $v(q) = 2\pi e^2/(\kappa q)$ with the background lattice dielectric constant of the sample κ , $F_{ij,i'j'}(q)$ is the subband form factor determined by the quantum confinement potential, and $\Pi_{ij}(q)$ are the polarizabilities corresponding to the two spin-split ($i, j = 1, 2$) subbands. Since suppression of screening by disorder scattering may be important (particularly at low densities) we include collisional broadening in the screening function through the Dingle temperature approximation [4].

We assume that the spin-split subbands have parabolic energy bands with spin degeneracy $g_s = 1$ (in each band) and isotropic Fermi surfaces with effective masses m_1 and m_2 . In our model the hole population of each subband, for a fixed total density $n = n_1 + n_2$, is determined by the effective mass ratio, i.e., $n_1/n_2 = m_1/m_2$, where m_i , n_i are the effective mass and population density for each group of carriers. Since there are two groups of holes we need to consider intersubband hole-hole scattering, which contributes to resistivity in addition to hole scattering by charged impurities. In particular, hole-hole scattering plays an important role when the carriers in each band have different masses and densities. In a two-subband system the resistivity without intersubband hole-hole scattering is given by $\rho(T) = 1/[\sigma_1(T) + \sigma_2(T)]$, where $\sigma_i = n_i e^2 \langle \tau_i \rangle / m_i$ is the individual conductivity of the i th subband. These τ_i can be calculated by using the usual Boltzmann transport equation, which becomes a

set of coupled transport equations for distribution functions associated with each subband [12].

The resistivity in the presence of hole-hole intersubband scattering of the two-band system is given by [9,10]

$$\rho(T) = \frac{\frac{n_2}{n_1}\sigma_1 + \frac{n_1}{n_2}\sigma_2 + \sigma}{\frac{n^2}{n_1 n_2}\sigma_1\sigma_2 + (\sigma_1 + \sigma_2)\sigma}, \quad (1)$$

where σ_i is the conductivity associated with the i th subband with the scattering times τ_i , and $\sigma = e^2 n \tau_{hh} m / m_1 m_2$ is the conductivity associated solely with hole-hole scattering with an average mass given by $m = (m_1 n_1 + m_2 n_2)/n$ and a hole-hole relaxation time (τ_{hh}) for the relative momentum of the two spin-split carrier systems. We now note an important feature of our spin-split two subband system: Spin in each subband is conserved and therefore, Coulomb scattering, being spin independent, cannot cause *real* intersubband scattering (which would change the spin index of the scattered carrier). Thus, only *intra-subband* transitions contribute to the intersubband hole-hole relaxation time, and we find

$$\frac{1}{\tau_{hh}} = \frac{2g_s^4(k_B T)^2}{3(2\pi)^2} mn \frac{m_1 m_2}{n_1 n_2} I(p), \quad (2)$$

where $g_s = 1$ is the subband degeneracy and $I(p) = p/(2\pi) \int_0^\pi d\theta \sin \theta W(q)[f(p)]^2$, where $f(p) = (1 + p^2 + 2p \cos \theta)^{1/2}$ with $p = (n_1/n_2)^{1/2}$ and $q = (4\pi/g_s)\sqrt{n_1 n_2} \sin(\theta)/f(p)$, and the collisional probability $W(q) = 2\pi |v(q)/\varepsilon(q)|^2$. In long wavelength Thomas-Fermi screening approximation (TFA) we have $W_{TFA}(q) = (2\pi)^3/[g_s(m_1 + m_2)]^2$ in our 2D system. In the absence of any real hole transitions from one subband to the other the RPA dielectric function is given by $\epsilon(q) = 1 - v(q)[\Pi_{11}(q) + \Pi_{22}(q)]$. The hole-hole scattering time τ_{hh} is symmetric with respect to the interchange of indices 1 and 2. Note that $\tau_{hh}^{-1} \propto T^2$ while $\tau_i^{-1} \propto T$ for $T/T_F \ll 1$ but the two asymptotic regimes need not coincide (additionally, the Dingle temperature suppresses temperature dependence for $T \ll T_D$), and therefore the net temperature dependence could be quite complex. When $\tau_{hh} \gg \tau_i$, which happens at very low temperatures, the total conductivity becomes the sum of the conductivities of each subband. When hole-hole scattering is very strong (or impurity scattering very weak), $\tau_{hh} \ll \tau_i$, Eq. (1) become $\rho(T) = (n_2^2 \sigma_1 + n_1^2 \sigma_1)/(n^2 \sigma_1 \sigma_2)$. Thus, in these two limits τ_{hh} makes negligible contribution to the system resistivity. The resistivity has a linear temperature dependence in these limits (provided $T \ll T_F$, and Dingle temperature effects are negligible). When $\tau_{hh} \sim \tau_i$ the temperature dependence of the resistivity is strongly affected by hole-hole scattering and becomes (for $T < T_F$) $\rho_T = \rho_0 + aT + bT^2$, where $\rho_0 = \rho(T=0)$ and a, b are positive density dependent constants. Our numerical calculation shows $a > b$ when $\tau_i > \tau_{hh}$ and $a < b$ when $\tau_i < \tau_{hh}$.

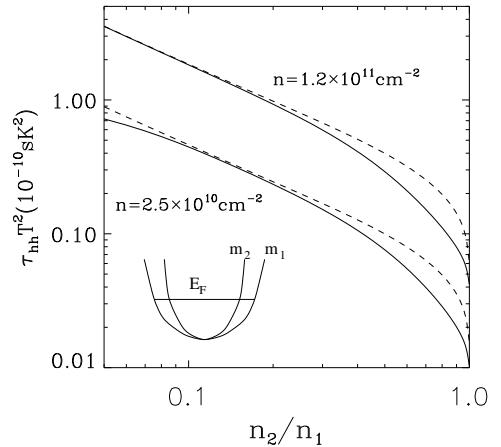


FIG. 1. Calculated hole-hole scattering time τ_{hh} as a function of the density ratio $r = n_2/n_1$ for fixed total densities $n = 1.2 \times 10^{11} \text{ cm}^{-2}$, $n = 2.5 \times 10^{10} \text{ cm}^{-2}$ at $T = 0.1 \text{ K}$. Solid (dashed) lines are the results from the screened Coulomb potential within the RPA (TFA). Inset shows the spin-split energy subbands.

In Fig. 1 we show the calculated hole-hole scattering time as a function of the density ratio $r = n_2/n_1$ for fixed total densities, $n = n_1 + n_2 = 2.5 \times 10^{10} \text{ cm}^{-2}$, $1.2 \times 10^{11} \text{ cm}^{-2}$. Inset shows schematically the spin-split energy subbands with different effective masses. We use a fixed effective mass $m_2 = 0.3m_e$ throughout this paper and create unequal population of the subbands (spin-splitting) by tuning $m_1 > m_2$. (Since Eq. (2) is symmetric with respect to the interchange of indices 1 and 2 we show the results for $n_2/n_1 < 1$). Fig. 1 shows that the scattering time of RPA is different from that of TFA when the density difference is small. Since most experimental situations [5,6] correspond to $0.5 < n_2/n_1 < 1$ the use of the proper screening function (i.e. RPA) is crucial (and long wavelength delta function scattering of TFA is inadequate), especially at low densities where the spin-splitting is very small. Fig. 1 also shows that at a given density and temperature the hole-hole scattering time increases as the population difference of the two bands increases, and the inverse scattering time τ_{hh}^{-1} diverges logarithmically as $r \rightarrow 1$. (This follows analytically from Eq. (2) also.) The scattering time has strong density dependence at a fixed temperature: the scattering time decreases as the density decreases.

The effect of changing the confining potential is shown in Figs. 2 and 3, where we show the calculated fractional change in the resistivity $\rho(T)/\rho(0)$ as a function of temperature. In Fig. 2 the results for a high density sample $n = 1.2 \times 10^{11} \text{ cm}^{-2}$ are given. The density ratio r decides the strength of the spin-splitting. (Smaller r indicates larger spin-splitting.) For $n_1 = n_2$ the fractional change in the resistivity shows a linear T dependence and is determined entirely by the mobilities of each band. In this case we have exactly the same results as the single band model with $g_s = 2$ and $n = n_1 + n_2$. When we tune

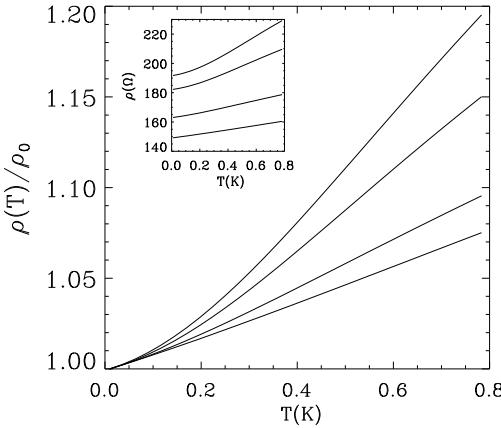


FIG. 2. Calculated fractional change in the resistivity $\rho(T)/\rho_0$ as a function of for $n = 1.2 \times 10^{11} \text{ cm}^{-2}$. Various curves correspond to $r = 1.0, 0.83, 0.67, 0.59$ (from bottom to top). The temperature dependent resistivity for a given density is shown in insets.

m_1 (keeping m_2 constant) the two subbands have unequal densities, $\Delta n = n_1 - n_2 = n(1 - r)/(1 + r)$. As r decreases the relative magnitude of the resistivity increases with temperature. Thus, the strength of the “metallicity” is enhanced when the two spin-split subbands have more unequal populations (i.e., larger spin splitting). This shows that hole-hole carrier scattering enhances the metallic behavior, particularly at high densities. (We get similar results for other high density samples.)

In Fig. 3 we show the results for a low density system, $n = 2.5 \times 10^{10} \text{ cm}^{-2}$. At this density experiment [5] shows that the lowest temperature resistivity, ρ_0 , is an order of magnitude larger than that at $n = 1.2 \times 10^{11} \text{ cm}^{-2}$. In addition, ρ_0 increases by about 100% with increasing back gate bias even though the total carrier density is constant. (Note that in the high density samples ρ_0 increases by $\leq 40\%$ with the same increase of the gate bias.) We believe that in low density systems the effective disorder becomes larger when the confining potential becomes asymmetric (due to the back gate bias). The important role of disorder in the 2D metallic behavior under a back gate bias has also recently been emphasized in Si inversion layer experiments [13]. To incorporate [3,4] the disorder induced collisional broadening corrections we introduce the Dingle temperature T_D in the screening function. The pure RPA ($T_D = 0$) case completely neglects collisional broadening effects on screening. The effect of the Dingle temperature is to increase the zero temperature resistivity (by suppressing screening for $T \ll T_D$) as the splitting becomes large. Fig. 3 shows that the temperature dependence of the resistivity is strongly enhanced when we consider a fixed Dingle temperature and hole-hole scattering. Without hole-hole scattering (and with a fixed Dingle temperature) the spin-split system still has a stronger metallic behavior than the corresponding one subband spin degenerate system ($r = 1$), but

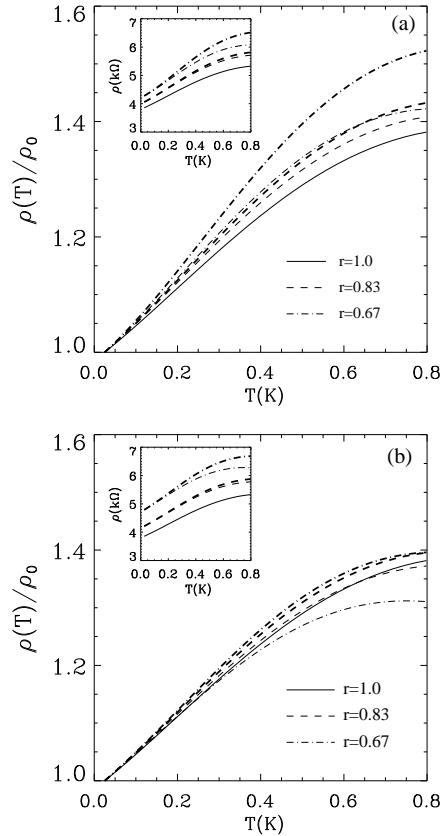


FIG. 3. Same as in Fig. 2 for a low density systems with $n = 2.5 \times 10^{10} \text{ cm}^{-2}$. Solid lines are for the equal density case ($n_1 = n_2$). Thin (thick) lines are the results without (with) hole-hole scattering. In (a) we use a fixed Dingle temperature ($T_D = 0.1T_F$) for different r , and in (b) we use different Dingle temperatures; $T_D = 0.1, 0.15, 0.2T_F$ for $r = 1.0, 0.83, 0.67$, respectively. The insets show $\rho(T)$ for the various case in the main figures.

the strength of the temperature dependence is reduced compared to the results with hole-hole scattering [Fig. 3(a)]. When we vary the Dingle temperature (i.e., increase the level broadening) the metallic behavior without hole-hole scattering decreases as the spin-splitting increases (Fig. 3(b)), which is precisely what is found experimentally [5]. Thus, without introducing the hole-hole scattering we can obtain qualitative agreement with experimental results for lower density samples including only disorder and temperature dependent screening.

It is unclear whether our model including only spin-splitting induced hole-hole scattering and screening induced temperature dependent disorder scattering can entirely quantitatively describe the metallic behavior observed in low density 2D hole systems. But it is clear that our zeroth order model should be an essential part of any theory in this subject. We have established beyond any reasonable doubt that both spin-splitting induced hole-hole scattering and temperature dependent screening play important roles in the observed “metallicity” of 2D p-GaAs systems.

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